Occasional Thoughts

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The Market's Social Welfare Function

We know that under the proper circumstances--no externalities, non-increasing returns, appropriate information about commodities, no market power, *et cetera*--the market system's equilibrium is a point on the economy's Pareto frontier. We also know that every point on the Pareto frontier is the maximum of some Social Welfare Function--some SWF. It is interesting and instructive to think about these two in conjunction, and to ask the question, "What is the market system's SWF? What SWF is the market considered as a social resource planning and commodity allocation mechanism maximizing?"

Let's consider a production economy with M final goods c, indexed 1…m…M, available in continuous quantities. Let's assume that there are J members of society, indexed 1…j… J, define U_j to be the utility of the jth member of society. Let's further assume that each member's utiltiy is a function of the quantities of the M goods allocated to that member by the market system's equilibrium.

Let's define a SWF as a weighted sum of individual utilities, with λ_k being the weight attached to the utility of individual k. And let's define the market system's SWF (the

MSSWF) as the set of λs and the corresponding weighted-sum function that is maximized at the market system's allocation, which we will assume is an internal one.

We can write the MSSWF as:

$$MSSWF = \sum_i \lambda_i U_i$$

And we can inquire into its properties. Let's consider a very small move along the Pareto frontier produced by the redistribution of a very small amount of some arbitrary and particular commodity, the mth, in an amount Δc_m from some member j to some member k. To first order, the effect of that redistribution on the MSSWF is:

$$\Delta MSSWF = \left[-\lambda_j \left(\frac{\partial U_j}{\partial c_m} \right)^* + \lambda_k \left(\frac{\partial U_k}{\partial c_m} \right)^* \right] \Delta c_m$$

where the *s indicate that these derivatives are evaluated at the market system's allocation. This redistribution is feasible to first order. And to first order this change in the MSSWF must be zero. If it is not zero, then the MSSWF is not the SWF that the market system is maximizing: you could raise the MSSWF by making a small enough transfer--either from j to k or from k to j:

$$0 = \left[-\lambda_j \left(\frac{\partial U_j}{\partial c_m} \right)^* + \lambda_k \left(\frac{\partial U_k}{\partial c_m} \right)^* \right] \Delta c_m$$

Which implies that:

$$\lambda_j \left(\frac{\partial U_j}{\partial c_m} \right)^* = \lambda_k \left(\frac{\partial U_k}{\partial c_m} \right)^*$$

Because the market's allocation is a competitive allocation and an internal allocation, each member is maximizing his or her utility given the market value of her endowment. That means that there is a number for each individual m which we will call the marginal utility of wealth, and this marginal utility of wealth is equal to the marginal utility of each commodity divided by the equilibrium price of that commodity:

$$\frac{\partial U_j}{\partial W} = \left(\frac{\partial U_j}{\partial c_m} \right)^* \frac{1}{p_m}$$

.

for all of the M commodities in the economy. We can then substitute in for the marginal utilities of commodity m in the equation two above:

$$\lambda_j \left(\frac{\partial U_j}{\partial W} \right) p_m = \lambda_k \left(\frac{\partial U_k}{\partial W} \right) p_m$$

Cancelling the common price terms on both sides, we see that the ratio of members' weights in the market system's SWF:

$$\frac{\lambda_j}{\lambda_k} = \frac{\left(\frac{\partial U_k}{\partial W} \right)}{\left(\frac{\partial U_j}{\partial W} \right)}$$

is inversely proportional to the ratio of members' marginal utilities of wealth, which was to be shown.